1- Double Integral

(1)Find the following integrals :

(i) $\int_0^1 \int_0^2 (12x\,y)\,dx\,dy$ (ii) $\int_0^1 \int_0^2 (12\,x^3y)\,dy\,dx$ (iii) $\int_0^1 \int_0^x (x+e^y)\,dy\,dx$

(iv) $\int_0^1 \int_0^{2y} (2x\,y)\,dx\,dy$ (v) $\int_1^2 \int_0^x (x^3-y)\,dy\,dx$ (vi) $\int_1^2 \int_1^x \frac{x}{y}\,dy\,dx$

(2)Find the following integrals:

(i) $\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ (ii) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ (iii) $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{x^2+y^2}\,dy\,dx$

(iv) $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{dx\,dy}{\sqrt{x^2+y^2}}$ (v) $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{x\,y}{\sqrt{x^2+y^2}}\,dx\,dy$ (vi) $\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{x+y}{\sqrt{x^2+y^2}}\,dy\,dx$

(3)Find the following integrals:

(i) $\iint_D \sqrt{x^2+y^2}\,dx\,dy$ (ii) $\iint_D \frac{\ln(x^2+y^2)}{x^2+y^2}\,dx\,dy$ (iii) $\iint_D \frac{\sin(x^2+y^2)}{x^2+y^2}\,dx\,dy$

Where D is the region between the two circles : $x^2+y^2=1,\ x^2+y^2=4.$

2- Special Functions

Find the following integrals:

(1) $\int_0^\infty e^{-\sqrt{x}}\,dx$ (2) $\int_0^\infty e^{-x^4}\,dx$ (3) $\int_0^1 x^2\,(-\ln x)^4\,dx$

(4) $\int_0^\infty x^5 e^{-2x^2}\,dx$ (5) $\int_2^\infty e^{4x-x^2}\,dx$ (6) $\int_3^\infty e^{-x^2+6x}\,dx$

(7) $\int_0^1 \sqrt[3]{1-x^3}\,dx$ (8) $\int_0^3 x\,(27-x^3)^{\frac{1}{3}}\,dx$ (9) $\int_0^{\frac{\pi}{2}} \sec x\,.\sqrt{\tan x}\,dx$

(10) $\int_3^4 (x-3)^4\sqrt{4-y}\,dy$ (11) $\int_0^2 z\,(16-z^4)^{\frac{1}{4}}\,dz$ (12) $\int_0^{\frac{\pi}{2}} \sin t\,.\sqrt{\tan t}\,dt$

(13)Express $\int J_4(x)dx$ in terms of $\int J_0(x)dx$

(14)Express $\int x.J_1(x)dx$ in terms of $\int J_0(x)dx$

(15)Express $\int x^2.J_0(x)dx$ in terms of $\int J_0(x)dx$

(16)Show that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\left(\frac{3}{x^2} - 1\right)\sin x - \frac{3}{x}\cos x\right]$

(17)Show that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3}{x}\sin x + \left(\frac{3}{x^2} - 1\right)\cos x\right]$

(18)Find the integrals: $(i)\ \int_0^1 x^2 J_0\, dx$ $(ii)\ \int_1^2 x^2 J_1\, dx$ $(iii)\ \int_0^1 x^3 J_3\, dx$

3- Partial Differential Equations (PDE)

(1)Solve the following PDE :

(i) $u_x + 3u_y - 2u = 0$ (ii) $2u_x + 3u_y - 4u = 0$

(iii) $u_x - u_y = 4u$ (iv) $2u_x + u_y - 3x = 0$

(v) $3u_x + 4u_y - 5u = 10y$ (vi) $4u_x + 3u_y - 10u = 5$

(2)Find the solution of each of the following PDE :

(i) $2u_x + 3u_y - u = 0,\quad u(x,0) = 4$

(ii) $3u_x - u_y + u = 0,\quad u(x,0) = 2e^{5x} + 3e^{-3x}$

(iii) $3u_x - u_y + u = 0,\quad u(0,y) = 3e^{5y}$

(iv) $2u_x + u_y + u = 0,\quad u(x,0) = 4e^{-2x}$

(3)Classify and solve the following PDE :

(i) $u_{xx} - 5u_{xy} + 6u_{yy} = 0$ (ii) $u_{xx} - 5u_{xy} + u_{yy} = e^{2x+3y}$

(iii) $u_{xx} - 2u_{xy} - 3u_{yy} = 0$ (iv) $u_{xx} - 5u_{xy} + 6u_{yy} = e^{3x+y}$

(v) $u_{xx} - u_x - 2u = 0$ (vi) $u_{yy} - 5u_y + 6u = 0$

(vii) $u_{xx} - u_x - 2u = e^{3x}$ (viii) $u_{yy} - 4u_y + 4u = e^{2y}$

(iv) $u_{xx} - 4u_{xy} + 4u_{yy} = xy^2$ (x) $u_{xx} - 4u_{xy} = \cos(2x + 3y)$

(xi) $u_{xx} + 3u_{yy} = 3x + y^2$ (xii) $u_{xx} - 4u_{xy} = \sin(4x + y)$

(4)Solve the following PDE :

(i) $u_{xx} - u_{xy} - 2u_{yy} + 2u_x + u_y - 5u = 0$ (ii) $u_{xx} + 3u_{xy} + 2u_{yy} + u_x - u_y + 6u = 0$

(iii) $u_{xx} - 4u_{xy} + u_x - 4u_y - u = 0$ (iv) $u_{xx} - 9u_{yy} - 2u_y + 4u = 0$

(5)Solve the following wave equations :

(i) $4u_{xx} = u_{tt}$, $0 < x < 3$

B.C: $u(0, t) = u(3, t) = 0$

I.C: $u(x, 0) = 5$, $u_t(x,0) = x$

(ii) $9u_{xx} = u_{tt}$, $0 < x < 2$

B.C: $u(0, t) = u(2, t) = 0$

I.C: $u(x,0) = x - 1$, $u_t(x,0) = 4x$

(iii) $25u_{xx} = u_{tt}$, $0 < x < 3$

B.C: $u(0, t) = u(3, t) = 0$

I.C: $u(x, 0) = x + 1$, $u_t(x,0) = x$

(iv) $4u_{xx} = 9u_{tt}$, $0 < x < 3$

B.C: $u(0, t) = u(3, t) = 0$

I.C: $u(x, 0) = x$, $u_t(x,0) = x$

(v) $9u_{xx} = u_{tt}$, $0 < x < \pi$

B.C: $u(0, t) = u(\pi, t) = 0$

I.C: $u(x, 0) = \sin x$, $u_t(x,0) = x$

(v) $9u_{xx} = 4u_{tt}$, $0 < x < \pi$

B.C: $u(0, t) = u(\pi, t) = 0$

I.C: $u(x, 0) = \cos x$, $u_t(x,0) = 2x$

(6)Solve the following heat equations :

(i) $u_t = u_{xx}$, $0 < x < 1$, $t > 0$.

B.C: $u(0, t) = u(1, t) = 0$, $t > 0$

I.C : $u(x, 0) = x + 1$, $0 < x < 1$.

(ii) $4u_t = u_{xx}$, $0 < x < 2$, $t > 0$.

B.C: $u(0, t) = u(2, t) = 0$, $t > 0$

I.C : $u(x, 0) = x - 1$, $0 < x < 2$.

(iii) $4u_t = 9u_{xx}$, $0 < x < 1$, $t > 0$.

B.C: $u(0, t) = u(1, t) = 0$, $t > 0$

I.C : $u(x, 0) = x + 2$, $0 < x < 1$.

(iv) $u_t = 16u_{xx}$, $0 < x < 4$, $t > 0$.

B.C: $u(0, t) = u(1, t) = 0$, $t > 0$

I.C : $u(x, 0) = 2x$, $0 < x < 4$.

(v) $u_t = 9u_{xx}$, $0 < x < \pi$, $t > 0$.

B.C: $u(0, t) = u(\pi, t) = 0$, $t > 0$

I.C : $u(x, 0) = \sin x$, $0 < x < \pi$.

(vi) $u_t = u_{xx}$, $0 < x < \pi/2$, $t > 0$.

B.C: $u(0, t) = u(\pi/2, t) = 0$, $t > 0$

I.C : $u(x, 0) = \cos x$, $0 < x < \pi/2$.

(7)Solve the following Laplace equations :

(i) $u_{xx} + u_{yy} = 0$, $0 < x < 1$, $0 < y < 1$.

B.C: $u(0, y) = u(1, y) = 0$.

I.C : $u(x, 0) = x$, $u(x, 1) = 0$.

(ii) $u_{xx} + u_{yy} = 0$, $0 < x < 2$, $0 < y < 1$.

B.C: $u(0, y) = u(2, y) = 0$.

I.C : $u(x, 0) = x$, $u(x, 1) = x$.

(iii) $u_{xx} + u_{yy} = 0$, $0 < x < 2$, $0 < y < 4$.

B.C: $u(0, y) = u(2, y) = 0$.

I.C : $u(x, 0) = x + 1$, $u(x, 4) = 0$.

(iv) $u_{xx} + u_{yy} = 0$, $0 < x < 1$, $0 < y < 2$.

B.C: $u(0, y) = u(1, y) = 0$.

I.C : $u(x, 0) = x$, $u(x, 2) = x + 2$.

(8)By Laplace transform, solve the following PDE :

(i) $u_x + u_t = x\,, \quad u(0,t) = 0,\ t \geq 0,\ u(x,0) = 0,\ x \geq 0.$

(ii) $u_{xx} + u_t = e^t\,, \quad u(0,t) = 0,\ t \geq 0,\ u(x,0) = 0,\ x \geq 0.$

(iii) $u_x + u_{tt} = t\,, \quad u(0,t) = 0,\ t \geq 0,\ u(x,0) = u_t(x,0) = 1,\ x \geq 0.$

(iv) $u_{xx} + u_{tt} = 1\,, \quad u(0,t) = 0,\ t \geq 0,\ u(x,0) = u_t(x,0) = 0,\ x \geq 0.$

(9)Solve the following initial boundary value problems :

(i) $u_{yy} + u_y = 0\,, \quad u(x,0) = f(x),\ u_x(x,0) = g(x).$

(ii) $u_{xx} + u_x = 0\,, \quad u(0,y) = \sin y,\ u_y(0,y) = e^y.$

4- Probability and Statistics

(1) Find $\bar{x}$, σ from the data :

x_i	2	3	4	5	6	7	8	9	10
f_i	3	5	8	12	15	11	9	6	4

(2) Find $\bar{x}$, σ from the data :

x_i	1	2	3	4	5	6	7	8	9	10
f_i	3	5	8	12	15	11	9	6	4	2

(3) Write the table of frequency and find $\bar{x}, \sigma$ from the data :

2, 2, 3, 4, 4, 4, 5, 5, 7, 7, 7, 7, 10, 10, 10

(4) Write the table of frequency and find $\bar{x}, \sigma$ from the data :

3, 3, 3, 4, 4, 5, 5, 5, 6, 6, 7, 7, 7, 10, 10, 10, 10, 10.

(5)Find the regression line of y on x and the regression line of x on y of the following data. Also, find $\overline{x}$, $\overline{y}$, σ_x, σ_y, $cov(x, y)$ and the correlation coefficient r.

(i) (2, 3), (4, 4), (6, 7), (8, 9), (10, 12)

(ii) (1, 2), (2, 3), (3, 5), (4, 8), (5, 9)

(iii) (1, 2), (1.5, 3), (2, 4.4), (2.5, 6), (3, 8)

(iv) (0.2, 1.4), (0.4, 2), (0.6, 2.5), (0.8, 3), (1, 4)

(v) (10, 1), (20, 3), (30, 4), (40, 5), (50, 8)

(6)Find the curves $y = a + b \ln x$, $y = ae^{bx}$, $y = a\, b^{x}$ and find the correlation coefficient r of each curve from the following data:

(i) (2, 3), (4, 4), (6, 7), (8, 9), (10, 12)

(ii) (1, 2), (2, 3), (3, 5), (4, 8), (5, 9)

(iii) (1, 2), (1.5, 3), (2, 4.4), (2.5, 6), (3, 8)

(iv) (0.2, 1.4), (0.4, 2), (0.6, 2.5), (0.8, 3), (1, 4)

(v) (10, 1), (20, 3), (30, 4), (40, 5), (50, 8)

(7) By Baye's theorem, find $P(A_1/A)$, $P(A_2/A)$, $P(A_3/A)$, the sample space, $S = \{3, 4, 5, 6, 7, 8, 9\}$, $A = \{3, 5, 8, 9\}$, $A_1 = \{3, 4\}, A_2 = \{5, 6\}$, $A_3 = \{7, 8, 9\}$.

(8) By Baye's theorem, find $P(A_1/A)$, $P(A_2/A)$, $P(A_3/A)$, the sample space $S = \{a, b, c, d, e, g\}$, $A = \{b, c, e, g\}$, $A_1 = \{a, b\}$, $A_2 = \{c\}$, $A_3 = \{d, e, g\}$.

(9)A box contains 10 balls, 4 red and 6 white. At random, one ball is selected with replacement. Find the probability that:

(a)The two selected balls are white.

(b)The second selection is white where the first is red.

(c)The second selected ball is white.

(10)A box contains 10 balls, 4 red and 6 white. At random, one ball is selected without replacement. Find the probability that:

(a)The two selected balls are white.

(b)The second selection is white where the first is red.

(c)The second selected ball is white.

(11) Write the table of frequency and the Pdf of the data :

(a) 2, 2, 3, 4, 4, 4, 5, 5, 7, 7, 7, 7, 10, 10, 10

(b) 3, 3, 3, 4, 4, 5, 5, 5, 6, 6, 7, 7, 7, 10, 10, 10, 10, 10.

(12)From the following discrete function Pdf :

x	$x < 1$	1	2	3	4	5	$x > 5$
$f(x)$	0	0.1	0.3	0.3	0.2	0.1	0

Find: (a) F(x) (b) $P(x < 2)$ (c) $P(1 < x \leq 4)$

(d) $P(x \leq 3)$ (e) $P(x < 4)$ (f) $P(1 \leq x \leq 4)$

(13)If x is random variable with discrete Pdf given by :

x	$x < 1$	1	2	3	4	$x > 4$
$f(x)$	0	0.1	0.4	0.3	0.2	0

Find:(a) F(x) (b) $P(x < 2)$ (c) $P(1 < x < 3)$

(d) $P(x < 2)$ (e) $P(x < 3)$ (f) $P(1 \leq x \leq 3)$

(14)A box contains 12 balls, 5 red and 7 white. At random, four balls are selected simultaneously. If x is the number of red balls, find the Pdf f(x).

(15)A box contains 15 balls, 4 red and 5 white and 6 blue. At random, three balls are selected simultaneously. If x is the number of red balls, find the Pdf f(x).

(16)A box contains 15 balls, 2 red and 5 white and 8 blue. At random, three balls are selected simultaneously. If x is the number of red balls, find the Pdf f(x).

(17)Show that the function : $f(x) = 2e^{-2x},\ 0 \le x < \infty$ is Pdf.

(18)If the following function is Pdf : $f(x) = \begin{cases} c\,e^{-2x}, & x \ge 0 \\ 0, & x < 0 \end{cases}$

Find : (a)The value of c (b) F(x)

(c) $P(x \le 2)$ (d) $P(x > 2)$

(e) $P(1 \le x \le 3)$ (f) $P(x \ge 0)$

(19)If the following function is Pdf : $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ c(3-x), & 1 < x \le 3 \\ 0, & otherwise \end{cases}$

Find : (a)The value of c (b) F(x)

(c) $P(x \le 0.5)$ (d) $P(x > 1)$

(e) $P(0.5 \le x \le 2)$ (f) $P(x \ge 2)$

(20)If $f(x) = 3e^{-3x},\ x \ge 0$ is Pdf of a random variable x. Find E(x).

(21)If $f(x) = \frac{5}{2}x^4,\ -1 \le x \le 1$ is Pdf of a random variable x. Find E(x)

(22)If the following function is Pdf : $f(x) = \begin{cases} c(1-x), & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$

Find : (a) c (b) F(x) (c) $\mu_x,\ v_x,\ \sigma_x$

(23)If x is random variable of discrete Pdf :

x	$x<1$	1	2	3	4	5	6	$x>6$
$f(x)$	0	0.12	0.18	0.25	0.2	0.15	0.10	0

Find μ, V, σ

(24)If x is random variable of discrete Pdf :

x	$x<2$	2	6	8	10	$x>10$
$f(x)$	0	0.2	0.35	0.3	0.15	0

Find μ, V, σ

(25)From the data, find $cov(x,y)$ and r.

x_i	2	3	5	7	9	9	11	14
y_i	5	6	8	11	12	14	14	5

(26)From the data, find $cov(x,y)$ and r.

x_i	2	3	5	7	8	9	10
y_i	5	6	8	11	12	13	15

(27)If x and y are random variables with join Pdf:

$f(x,y) = \frac{1}{42}(x+2y),\ x = 1,2,3\ and\ y = 1,3,4.$

Write the table of the Pdf and find $P(x=1, y=3), P(x \le 2, y \le 3)$,

$P(x \ge 1, y \le 2)$, μ_x, μ_y, V_x, V_y, σ_x, σ_y, $cov(x,y)$ and r.

(28)If x and y are random variables with join Pdf:

$f(x,y) = \frac{2}{55}(x+y^2),\ x = 0,1,2\ and\ y = 1,1.5,2$

Write the table of the Pdf and find $P(x=1,\ y=1), P(x<2,\ y\le 1)$,

$P(x\le 2,\ y\le 1), P(x\ge 1,\ y\le 1.5)$, μ_x, μ_y, V_x, V_y, σ_x, σ_y, $cov(x,y)$ and r.

(29)If $f(x) = 2e^{-2x},\ x \ge 0$ is Pdf.

Find $M_x(t)$ and $m_0,\ m_1,\ m_2, m_4,\ \mu_0,\ \mu_1,\ \mu_2$

(30)If $f(x) = x^2e^{-x},\ x \ge 0$ is Pdf.

Find $M_x(t)$ and $m_0,\ m_1,\ m_2, m_3,\ \mu_0,\ \mu_1,\ \mu_2$

(31)If x is random variable with discrete Pdf given by :

x	$x<2$	2	3	4	5	$x>5$
$f(x)$	0	0.1	0.3	0.4	0.2	0

Find $m_0,\ m_1,\ m_2,\ \mu_0,\ \mu_1,\ \mu_2$.

(32)A box contains 8 balls, 3 are red and 5 are white. At random, two balls are drawn. If x is the number of red balls. Find

(i)The Pdf $f(x)$. (ii)The probability distribution $F(x)$

(iii)Find $m_0,\ m_1,\ m_2,\ V,\ \sigma$.

(33)A box contains 9 balls, 3 are red and 6 are white. At random, three balls are drawn. If x is the number of red balls. Find

(i)The Pdf $f(x)$. (ii)The probability distribution $F(x)$

(iii)Find $m_0,\ m_1,\ m_2,\ V,\ \sigma$.

(34)Find the value of c such that $f(x) = \begin{cases} c(1-x), & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$

is Pdf. Also, $M_x(t),\ \mu\ and\ V$.

(35)If x and y are random variables with join Pdf $f(x,y) = e^{-x-y}$,

$x, y \ge 0$. Find $\mu_x,\ \mu_y,\ V_x,\ V_y,\ \sigma_x,\ \sigma_y$ and cov(x, y).

(36)If x and y are random variables with join Pdf $f(x,y) = 2e^{-2x-y}$,

$x, y \ge 0$. Find $\mu_x,\ \mu_y,\ V_x,\ V_y,\ \sigma_x,\ \sigma_y$ and cov(x, y).

(37)If x and y are random variables with join Pdf $f(x,y) = 4xy$,

$0 \le x, y \le 1$. Find $\mu_x,\ \mu_y,\ V_x,\ V_y,\ \sigma_x,\ \sigma_y$ and cov(x, y).

(38)In rolling a dice 5 times. Find the probability of appearing number 6 at least 3 times.

(39)If the probability of producing a defective screw is p = 0.02.

Find the probability that a lot of 100 screw contains more than 3 defectives.

(40)Three coins are tossed simultaneously and x is the number of heads. Find the probabilities, head = H and tail = T :

(a) No H appear (b) One H appears

(c) At least one H appears (d) Not more than three H appear

(41)From the table of normal distribution. Find

(a) $P(x \le 1.5)$ (b) $P(x < 2.8)$ (c) $P(1 < x \le 4)$

(d) $P(x \ge 2)$ (e) $P(x > 2.8)$ (f) $P(3 \le x \le 5)$

(42)From the normal distribution, $\mu = 0.5\ and\ \sigma = 1$. Find

(a) $P(x \le 2)$ (b) $P(x < 2.44)$ (c) $P(1 \le x \le 1.8)$

(d) $P(x \ge 1)$ (e) $P(x > 2)$ (f) $P(2 \le x \le 3)$

(43)From the normal distribution, $\mu = 4\ and\ \sigma = 0.3$

Find : (a) the value of k such that $P(x \le k) = 0.96$

(b) the value of k such that $P(x \ge k) = 0.02$

(44)In a production of iron rods, if the diameter x is normally distributed with $\mu = 2 \ and \ \sigma = 0.002.$

Find : (a) the percentage of defectives with tolerance limits 2 ± 0.01

(b) the tolerance limits to allow defectives 0.03.

(45)From Gamma distribution, when n = 2, find μ, V, σ and

(a) $P(x \leq 0.5)$ (b)$P(x < 1)$ (c) $P(x \leq 2)$

(d) $P(x \leq 8)$ (e) $P(x > 2)$ (f) $P(x \geq 5)$

(46)From Gamma distribution, when n = 3, find μ, V, σ and

(a) $P(x \leq 0.25)$ (b)$P(x < 1.5)$ (c) $P(x \leq 3)$

(d) $P(x \geq 2)$ (e) $P(x > 5)$ (f) $P(x \geq 8)$

(47)From Beta distribution, when m = 4, n = 3, find μ, V, σ and

(a) $P(x \leq 0.2)$ (b) $P(x < 0.5)$ (c) $P(x \leq 0.9)$

(d) $P(x \leq 1)$ (d) $P(x > 0.5)$ (d) $P(x \geq 0.8)$

(48)From Beta distribution, when m = 2, n = 3, find μ, V, σ and

(a) $P(x \leq 0.1)$ (b) $P(x < 0.8)$ (c) $P(x \leq 0.9)$

(d) $P(x \geq 0.2)$ (d) $P(x > 0.5)$ (d) $P(x \geq 0.7)$

Table A5 Binomial Distribution

Probability function $f(x)$ [see (2), Sec. 24.7] and distribution function $F(x)$

n	x	$p = 0.1$		$p = 0.2$		$p = 0.3$		$p = 0.4$		$p = 0.5$	
		$f(x)$	$F(x)$	$f(x)$	$F(x)$	$f(x)$	$F(x)$	$f(x)$	$F(x)$	$f(x)$	$F(x)$
1	0	**0.** 9000	0.9000	**0.** 8000	0.8000	**0.** 7000	0.7000	**0.** 6000	0.6000	**0.** 5000	0.5000
	1	1000	1.0000	2000	1.0000	3000	1.0000	4000	1.0000	5000	1.0000
2	0	8100	0.8100	6400	0.6400	4900	0.4900	3600	0.3600	2500	0.2500
	1	1800	0.9900	3200	0.9600	4200	0.9100	4800	0.8400	5000	0.7500
	2	0100	1.0000	0400	1.0000	0900	1.0000	1600	1.0000	2500	1.0000
3	0	7290	0.7290	5120	0.5120	3430	0.3430	2160	0.2160	1250	0.1250
	1	2430	0.9720	3840	0.8960	4410	0.7840	4320	0.6480	3750	0.5000
	2	0270	0.9990	0960	0.9920	1890	0.9730	2880	0.9360	3750	0.8750
	3	0010	1.0000	0080	1.0000	0270	1.0000	0640	1.0000	1250	1.0000
4	0	6561	0.6561	4096	0.4096	2401	0.2401	1296	0.1296	0625	0.0625
	1	2916	0.9477	4096	0.8192	4116	0.6517	3456	0.4752	2500	0.3125
	2	0486	0.9963	1536	0.9728	2646	0.9163	3456	0.8208	3750	0.6875
	3	0036	0.9999	0256	0.9984	0756	0.9919	1536	0.9744	2500	0.9375
	4	0001	1.0000	0016	1.0000	0081	1.0000	0256	1.0000	0625	1.0000
5	0	5905	0.5905	3277	0.3277	1681	0.1681	0778	0.0778	0313	0.0313
	1	3281	0.9185	4096	0.7373	3602	0.5282	2592	0.3370	1563	0.1875
	2	0729	0.9914	2048	0.9421	3087	0.8369	3456	0.6826	3125	0.5000
	3	0081	0.9995	0512	0.9933	1323	0.9692	2304	0.9130	3125	0.8125
	4	0005	1.0000	0064	0.9997	0284	0.9976	0768	0.9898	1563	0.9688
	5	0000	1.0000	0003	1.0000	0024	1.0000	0102	1.0000	0313	1.0000
6	0	5314	0.5314	2621	0.2621	1176	0.1176	0467	0.0467	0156	0.0156
	1	3543	0.8857	3932	0.6554	3025	0.4202	1866	0.2333	0938	0.1094
	2	0984	0.9841	2458	0.9011	3241	0.7443	3110	0.5443	2344	0.3438
	3	0146	0.9987	0819	0.9830	1852	0.9295	2765	0.8208	3125	0.6563
	4	0012	0.9999	0154	0.9984	0595	0.9891	1382	0.9590	2344	0.8906
	5	0001	1.0000	0015	0.9999	0102	0.9993	0369	0.9959	0938	0.9844
	6	0000	1.0000	0001	1.0000	0007	1.0000	0041	1.0000	0156	1.0000
7	0	4783	0.4783	2097	0.2097	0824	0.0824	0280	0.0280	0078	0.0078
	1	3720	0.8503	3670	0.5767	2471	0.3294	1306	0.1586	0547	0.0625
	2	1240	0.9743	2753	0.8520	3177	0.6471	2613	0.4199	1641	0.2266
	3	0230	0.9973	1147	0.9667	2269	0.8740	2903	0.7102	2734	0.5000
	4	0026	0.9998	0287	0.9953	0972	0.9712	1935	0.9037	2734	0.7734
	5	0002	1.0000	0043	0.9996	0250	0.9962	0774	0.9812	1641	0.9375
	6	0000	1.0000	0004	1.0000	0036	0.9998	0172	0.9984	0547	0.9922
	7	0000	1.0000	0000	1.0000	0002	1.0000	0016	1.0000	0078	1.0000
8	0	4305	0.4305	1678	0.1678	0576	0.0576	0168	0.0168	0039	0.0039
	1	3826	0.8131	3355	0.5033	1977	0.2553	0896	0.1064	0313	0.0352
	2	1488	0.9619	2936	0.7969	2965	0.5518	2090	0.3154	1094	0.1445
	3	0331	0.9950	1468	0.9437	2541	0.8059	2787	0.5941	2188	0.3633
	4	0046	0.9996	0459	0.9896	1361	0.9420	2322	0.8263	2734	0.6367
	5	0004	1.0000	0092	0.9988	0467	0.9887	1239	0.9502	2188	0.8555
	6	0000	1.0000	0011	0.9999	0100	0.9987	0413	0.9915	1094	0.9648
	7	0000	1.0000	0001	1.0000	0012	0.9999	0079	0.9993	0313	0.9961
	8	0000	1.0000	0000	1.0000	0001	1.0000	0007	1.0000	0039	1.0000

Table A7 Normal Distribution

Values of the distribution function $\Phi(z)$ [see (3), Sec. 24.8]. $\Phi(-z) = 1 - \Phi(z)$

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
	0.		**0.**		**0.**		**0.**		**0.**		**0.**
0.01	5040	0.51	6950	1.01	8438	1.51	9345	2.01	9778	2.51	9940
0.02	5080	0.52	6985	1.02	8461	1.52	9357	2.02	9783	2.52	9941
0.03	5120	0.53	7019	1.03	8485	1.53	9370	2.03	9788	2.53	9943
0.04	5160	0.54	7054	1.04	8508	1.54	9382	2.04	9793	2.54	9945
0.05	5199	0.55	7088	1.05	8531	1.55	9394	2.05	9798	2.55	9946
0.06	5239	0.56	7123	1.06	8554	1.56	9406	2.06	9803	2.56	9948
0.07	5279	0.57	7157	1.07	8577	1.57	9418	2.07	9808	2.57	9949
0.08	5319	0.58	7190	1.08	8599	1.58	9429	2.08	9812	2.58	9951
0.09	5359	0.59	7224	1.09	8621	1.59	9441	2.09	9817	2.59	9952
0.10	5398	0.60	7257	1.10	8643	1.60	9452	2.10	9821	2.60	9953
0.11	5438	0.61	7291	1.11	8665	1.61	9463	2.11	9826	2.61	9955
0.12	5478	0.62	7324	1.12	8686	1.62	9474	2.12	9830	2.62	9956
0.13	5517	0.63	7357	1.13	8708	1.63	9484	2.13	9834	2.63	9957
0.14	5557	0.64	7389	1.14	8729	1.64	9495	2.14	9838	2.64	9959
0.15	5596	0.65	7422	1.15	8749	1.65	9505	2.15	9842	2.65	9960
0.16	5636	0.66	7454	1.16	8770	1.66	9515	2.16	9846	2.66	9961
0.17	5675	0.67	7486	1.17	8790	1.67	9525	2.17	9850	2.67	9962
0.18	5714	0.68	7517	1.18	8810	1.68	9535	2.18	9854	2.68	9963
0.19	5753	0.69	7549	1.19	8830	1.69	9545	2.19	9857	2.69	9964
0.20	5793	0.70	7580	1.20	8849	1.70	9554	2.20	9861	2.70	9965
0.21	5832	0.71	7611	1.21	8869	1.71	9564	2.21	9864	2.71	9966
0.22	5871	0.72	7642	1.22	8888	1.72	9573	2.22	9868	2.72	9967
0.23	5910	0.73	7673	1.23	8907	1.73	9582	2.23	9871	2.73	9968
0.24	5948	0.74	7704	1.24	8925	1.74	9591	2.24	9875	2.74	9969
0.25	5987	0.75	7734	1.25	8944	1.75	9599	2.25	9878	2.75	9970
0.26	6026	0.76	7764	1.26	8962	1.76	9608	2.26	9881	2.76	9971
0.27	6064	0.77	7794	1.27	8980	1.77	9616	2.27	9884	2.77	9972
0.28	6103	0.78	7823	1.28	8997	1.78	9625	2.28	9887	2.78	9973
0.29	6141	0.79	7852	1.29	9015	1.79	9633	2.29	9890	2.79	9974
0.30	6179	0.80	7881	1.30	9032	1.80	9641	2.30	9893	2.80	9974
0.31	6217	0.81	7910	1.31	9049	1.81	9649	2.31	9896	2.81	9975
0.32	6255	0.82	7939	1.32	9066	1.82	9656	2.32	9898	2.82	9976
0.33	6293	0.83	7967	1.33	9082	1.83	9664	2.33	9901	2.83	9977
0.34	6331	0.84	7995	1.34	9099	1.84	9671	2.34	9904	2.84	9977
0.35	6368	0.85	8023	1.35	9115	1.85	9678	2.35	9906	2.85	9978
0.36	6406	0.86	8051	1.36	9131	1.86	9686	2.36	9909	2.86	9979
0.37	6443	0.87	8078	1.37	9147	1.87	9693	2.37	9911	2.87	9979
0.38	6480	0.88	8106	1.38	9162	1.88	9699	2.38	9913	2.88	9980
0.39	6517	0.89	8133	1.39	9177	1.89	9706	2.39	9916	2.89	9981
0.40	6554	0.90	8159	1.40	9192	1.90	9713	2.40	9918	2.90	9981
0.41	6591	0.91	8186	1.41	9207	1.91	9719	2.41	9920	2.91	9982
0.42	6628	0.92	8212	1.42	9222	1.92	9726	2.42	9922	2.92	9982
0.43	6664	0.93	8238	1.43	9236	1.93	9732	2.43	9925	2.93	9983
0.44	6700	0.94	8264	1.44	9251	1.94	9738	2.44	9927	2.94	9984
0.45	6736	0.95	8289	1.45	9265	1.95	9744	2.45	9929	2.95	9984
0.46	6772	0.96	8315	1.46	9279	1.96	9750	2.46	9931	2.96	9985
0.47	6808	0.97	8340	1.47	9292	1.97	9756	2.47	9932	2.97	9985
0.48	6844	0.98	8365	1.48	9306	1.98	9761	2.48	9934	2.98	9986
0.49	6879	0.99	8389	1.49	9319	1.99	9767	2.49	9936	2.99	9986
0.50	6915	1.00	8413	1.50	9332	2.00	9772	2.50	9938	3.00	9987

Table A2 **Gamma Function** [see (24) in App. A3.1]

α	$\Gamma(\alpha)$	α	$\Gamma(\alpha)$	α	$\Gamma(\alpha)$	α	$\Gamma(\alpha)$	α	$\Gamma(\alpha)$
1.00	1.000 000	1.20	0.918 169	1.40	0.887 264	1.60	0.893 515	1.80	0.931 384
1.02	0.988 844	1.22	0.913 106	1.42	0.886 356	1.62	0.895 924	1.82	0.936 845
1.04	0.978 438	1.24	0.908 521	1.44	0.885 805	1.64	0.898 642	1.84	0.942 612
1.06	0.968 744	1.26	0.904 397	1.46	0.885 604	1.66	0.901 668	1.86	0.948 687
1.08	0.959 725	1.28	0.900 718	1.48	0.885 747	1.68	0.905 001	1.88	0.955 071
1.10	0.951 351	1.30	0.897 471	1.50	0.886 227	1.70	0.908 639	1.90	0.961 766
1.12	0.943 590	1.32	0.894 640	1.52	0.887 039	1.72	0.912 581	1.92	0.968 774
1.14	0.936 416	1.34	0.892 216	1.54	0.888 178	1.74	0.916 826	1.94	0.976 099
1.16	0.929 803	1.36	0.890 185	1.56	0.889 639	1.76	0.921 375	1.96	0.983 743
1.18	0.923 728	1.38	0.888 537	1.58	0.891 420	1.78	0.926 227	1.98	0.991 708
1.20	0.918 169	1.40	0.887 264	1.60	0.893 515	1.80	0.931 384	2.00	1.000 000